Optimization of a Supersonic Panel Subject to a Flutter Constraint—A Finite-Element Solution

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Nomenclature

$[A_e]$ = basic element aerodynamic matrix
$[A]$ = system aerodynamic matrix
b = L/n = length of panel element
D_0, D_i = reference uniform panel stiffness, stiffness of ith element, Eq. (7)
$\{F\}_i$ = nodal forces on ith element, Eq. (1)
g_α = aerodynamic damping coefficient, Eq. (3)
k = an eigenvalue, Eq. (2)
$[K_e]$ = basic element stiffness matrix
$[K]$ = system stiffness matrix
L = length of panel
m_0 = uniform panel mass
m_s, m_f = structural mass, fixed (core) mass, Eq. (5)
m_i = mass of ith panel, Eq. (6)
$[M_e]$ = basic element mass matrix
$[M]$ = system mass matrix
n = number of elements
q = dynamic pressure
$\{q\}_i$ = nondimensional nodal displacement vector of ith element
U = freestream velocity
W = total structural weight fraction, Eq. (11)
α = a dynamic pressure parameter = λ/n^3
γ = an eigenvalue, Eq. (8)
δ = an eigenvalues = $k/420n^4$
η = structural mass fraction, Eq. (5)
λ_{cr} = dynamic pressure parameter = $2qL^3/D(M_\infty{}^2-1)^{1/2}$
λ_0 = critical (flutter) value of λ for uniform panel composed of n elements
ρ_i = ratio of structural mass of ith element to structural mass of uniform panel, Eq. (5)
ω_r = reference natural frequency, Eq. (3)

Introduction

THE finite-element method has been applied to supersonic panel flutter problems[1,2] through the derivation of what may be called a consistent aerodynamic matrix, i.e., an aerodynamic matrix based on the same interpolation functions and same coordinates as are used in deriving the stiffness and the mass matrices. Turner[3] extended his earlier work[4] by minimizing the mass of a system subject to a flutter constraint and used Olson's[1] panel equations in an example.

The present paper employs a direct approach to aeroelastic optimization based on gradient-projection techniques using a finite-element model of the structure.

Finite-Element Formulation of Equation of Motion

The panel of span L is taken to be of constant depth and to be divided into n equal-length elements in the spanwise direction. Panel bending stiffness and density are constant within each element. The resulting equation of motion for the element is[1]

$$\{F\}_i = D\big[[K_e]+\alpha[A_e]-\delta[M_e]\big]\{q\} \tag{1}$$

The element stiffness, mass, and aerodynamic matrices appear in Refs. 1 and 5. The time dependence of the displacements is taken to be $e^{\nu t}$ and

$$k = 420n^4\delta = -\lambda[(M_\infty{}^2-2)/(M_\infty{}^2-1)](L/U)\nu-(mL^4/D)\nu^2 \tag{2}$$

or

$$k = -g_\alpha(\nu/\omega_r)-(\nu/\omega_r)^2 \tag{3}$$

The elements are assembled by the direct stiffness method and boundary conditions are imposed to obtain the system equations. In the absence of nodal forces, $\{F\}$, an eigenvalue problem results. It is

$$\big[[K]+\alpha[A]-\delta[M]\big]\{q\} = \{0\} \tag{4}$$

where $\{q\}$ contains all unconstrained degrees of freedom. For a uniform panel Olson[1] has shown that the finite-element solution converges to the exact solution rapidly. For $n = 4$ there is only a 0.3% error in λ_{cr} (exact $\lambda_{cr} = 343.356$) and an 0.8% error in k_{cr}.

In order to treat panel flutter problems for sandwich panels or other panels having distributed nonstructural mass it is assumed that the mass per unit area is divided into a structural mass and a nonstructural (fixed) mass such that, for a uniform panel

$$m_0 = m_s+m_f = m_0\eta+m_0(1-\eta) \tag{5}$$

(Subscript 0 refers to uniform panel.) The structural mass may be varied. The mass ratios, ρ_i are the design parameters, and the total mass of the ith panel is thus given by

$$m_i = m_0(\rho_i\eta+1-\eta) \tag{6}$$

It is assumed that the stiffness is given by

$$D_i = \rho_i D_0 \tag{7}$$

as would be the case for thin face sheets separated by a nonstructural core.

In the remainder of this paper the aerodynamic damping will be neglected, i.e., $g_\alpha = 0$ will be assumed. Then, let

$$\gamma = \nu^2/420n^4\omega_r{}^2 \tag{8}$$

The element equation of motion becomes

$$\{F\}_i = D_0\big[\rho_i[K_e]+\alpha[A_e]+\gamma(\rho_i\eta+1-\eta)[M_e]\big]\{q\}_i \tag{9}$$

and the corresponding system eigenvalue equation becomes

$$\big[[K]+\alpha[A]+\gamma[M]\big]\{q\} = \{0\} \tag{10}$$

A Gradient Projection Optimization Procedure

The optimization problem treated here may be stated as follows: Minimize the total weight of the panel subject to the constraint $\lambda_{cr} = \lambda_0 =$ const. That is,

$$\text{Minimize } W \equiv \sum_{i=1}^{n} \rho_i \tag{11}$$

subject to

$$\lambda_{cr} \equiv \lambda_{cr}(\rho_i,\ldots,\rho_n) = \lambda_0 \tag{12}$$

where λ_{cr} is the maximum value of λ for which steady-state motion occurs.

A gradient-projection method[6] will be employed. A design step consists of moving a small distance ds, defined in the Euclidean sense by

$$ds^2 = \sum_{i=1}^{n} d\rho_i{}^2 \tag{13}$$

The steepest descent method would choose design variable changes to maximize the reduction in W for a given step size ds. The flutter constraint, Eq. (12), may be adjoined by the Lagrange multiplier technique to give the following equation for the appropriate design variable changes[5]

$$\Delta\rho_i = -c_1\left[\sum_{j=1}^{n}\left(\frac{\partial\lambda_{cr}}{\partial\rho_j}\right)^2 - \frac{\partial\lambda_{cr}}{\partial\rho_i}\sum_{j=1}^{n}\left(\frac{\partial\lambda_{cr}}{\partial\rho_j}\right)\right] \tag{14}$$

where c_1 is a positive constant selected so that the distance moved in the design space is "small."

Equation (14) forms the basis for what will be called the "weight-minimization mode" of optimization. In the present

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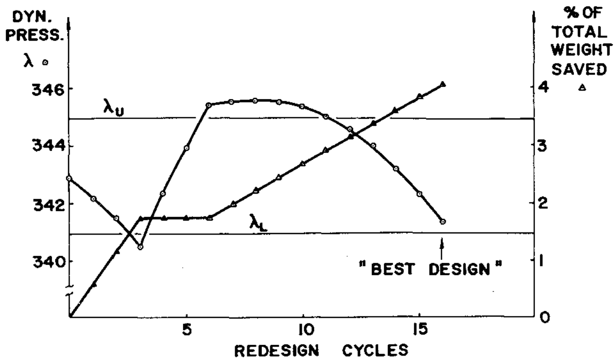

Fig. 1 Design cycles for 5-element panel; ($n = 5, \eta = 0.8, \lambda_u = \lambda_0 + 2.0, \lambda_L = \lambda_0 - 2.0$).

work the derivatives $\partial\lambda_{cr}/\partial\rho_i$ were evaluated numerically by incrementing successive values of ρ_i. However, other techniques[7] for evaluating derivatives will be used in future studies in order to reduce solution times.

Since the constraint equation is not linear in the design parameters, the direction of $\nabla\lambda_{cr}$, the gradient of λ_{cr}, is a function of the ρ_i's. In carrying out computations in the weight-minimization mode using Eq. (14) it was found that the value of λ_{cr} for the new design was not exactly λ_0. Hence, the following procedure was employed: the weight-minimization mode was employed until $\lambda_{cr}(\rho)$ fell below a specified value. At this point a "lambda-modification mode" was entered in which the weight was held constant at the last value attained by weight minimization, while the value of λ_{cr} was increased along the direction of steepest ascent subject to the weight constraint. The following equation governs the lambda-modification mode[5]:

$$\Delta\rho_i = c_2\left[\frac{\partial\lambda_{cr}}{\partial\rho_i} - \frac{1}{n}\sum_{j=1}^{n}\left(\frac{\partial\lambda_{cr}}{\partial\rho_j}\right)\right] \tag{15}$$

where c_2 is a positive constant.

Numerical Results

A computer program was written in which the element stiffness, aerodynamic, and mass matrices were generated according to Eq. (9) for a specified structural mass fraction, η, and a specified set of design parameters, ρ_i. These were assembled to form an eigenvalue problem of the form given by Eq. (10). The value of α was systematically varied until the critical value for the given design was obtained. Equations (14) and (15) were employed to determine design changes for weight-minimization steps and for lambda-modification steps, respectively.

The results shown in Figs. 1 and 2 represent improved designs and not optimal designs and represent only preliminary studies of the use of the weight-minimization, lambda-modification procedure. Figure 1 is for a panel with structural mass fraction $\eta = 0.8$ and having five finite elements, i.e., $n = 5$. The value of λ_0, as determined for $n = 5$, was $\lambda_0 = 342.901$.

In Fig. 1 weight-minimization proceeded until λ_{cr} fell below a limit set at $\lambda_L = \lambda_0 - 2.0$. Lambda-modification was allowed to raise λ_{cr} above λ_0 with an upper limit of $\lambda_u = \lambda_0 + 2.0$. It is seen that a weight reduction of 3.6% of the total weight was possible with $\lambda_{cr} = 343.190$. The best design gives a total weight reduction of 4.0% with $\lambda_{cr} = 341.315$. Reference 5 contains similar figures for a 5-element panel with $\lambda_u = \lambda_0$ and for a 9-element panel. The 9-element problem was terminated after four weight-minimization steps with no lambda-modification. Figure 2 shows the shape of the best designs for the 5-element and 9-element panels.

Although it might be possible to obtain better designs than the best design in Fig. 1, it is significant to note that the percent weight saved is considerably more than was obtained by Turner,[3] and the shape of the best designs shown in Fig. 2 indicate that Turner's use of $n = 3$ did not allow the design to assume a form resembling an optimum design.

Weisshaar[8] recently published results of his work on the same problem treated here. He employed tapered elements and used steepest descent directly without constraints. Using a 4-element model he shows an 8.5% weight saving after 4 design cycles, which were apparently carried out in an interactive mode. The shape of the optimized panel obtained by Weisshaar is in agreement with the shapes shown in Fig. 2 of the present paper.

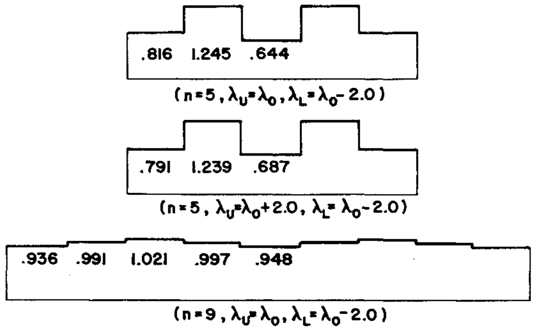

Fig. 2 Best design (m_i/m_0) for 5-element and 9-element panels.

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Elimination of Degrees of Freedom by Structural Splining

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IT is shown that a method of eliminating degrees of freedom presented by Guyan[1] is equivalent to interpolation by splining by means of the actual structure.

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